P425/2
APPLIED MATHEMATICS
Paper 2
July /Aug. 2024
3 hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in section A and any* **five** *from section B.*

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section B will **not** *be marked.*

Turn Over

SECTION A (40 MARKS)

1. Given that $P(A) = 0.7,\ P(A' \cup B) = 0.6,$ find
   (a) $P(A \cap B')$
   (b) $P(B/A)$ **(05 marks)**

2. A particle executing simple harmonic motion starts from rest and next comes to rest after 6 seconds, covering 10m in this time;
   Calculate the;
   (a) period of motion. **(02 marks)**
   (b) maximum speed of the particle. **(03 marks)**

3. Calculate the maximum error in the function $x\sin x$ at $x = 30^o \pm 0.5^o$ **(05 marks)**

4. Study the table below;

x	5	6	7	8	9	10
f	1	5	10	8	4	2

Using an assumed mean, m = 7, calculate the mean and variance of x. **(05 marks)**

5. The position vectors of two bodies A and B after t seconds of motion are;
   $r_A = \begin{pmatrix}8\\0\end{pmatrix} + t\begin{pmatrix}1\\8\end{pmatrix}$ and $r_B = \begin{pmatrix}0\\6\end{pmatrix} + t\begin{pmatrix}5\\5\end{pmatrix}$ metres. Calculate the shortest distance between the bodies, and state the value of t at which this occurs. **(05 marks)**

6. A box contains 3 red and 2 white balls of the same size. The balls are thoroughly mixed. Balls are picked one at a time with replacement. Ten balls are picked, find the probability that;
   (a) exactly 5,
   (b) over 8 balls are red. **(05 marks)**

7. The points in the table below were taken from the curve $y = x^2$

x	0.1	0.2
y	0.01	0.04

Calculate the absolute errors made in using linear interpolation or extrapolation to estimate;
   (a) y when $x = 0.18$
   (b) x when $y = 0.05$ **(05 marks)**

8. A particle is projected vertically upwards with a speed of $49ms^{-1}$. Calculate the time interval within which the particle is at least 78.4 metres above the ground. *(05 marks)*

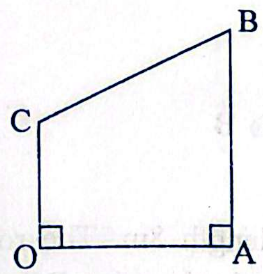
SECTION B (60 MARKS)

9. (a) Show that the equation $e^x = 2 - x$ has a root lying between 0 and 1. Hence use a graphical method to find the first approximate, x_o, to the equation. *(06 marks)*

(b) Use the Newton – Raphson method to find the root correct to 4 decimal places. *(06 marks)*

10. (a) ABCD is a uniform rectangular tray of mass 1kg, at rest on a horizontal table. Masses of 2, 4, 5 and 8kg are placed at the corners A, B, C and D respectively. Find the position of the centre of gravity of the system, with respect to the sides AB and AD. *(04 marks)*

(b)



OABC is a uniform lamina, in which $\overline{OA} = 6cm$, $\overline{AB} = 7cm$ and $\overline{OC} = 4cm$.

(i) Find the coordinates of the centre of gravity of the lamina, taking $\overline{OA}$ as the x – axis, and $\overline{OC}$ as the y – axis. *(04 marks)*

(ii) The lamina is suspended from point C, calculate the angle side OA makes with the vertical. *(04 marks)*

11. The table below shows the distribution of marks obtained by students in a paper 2 examination.

Marks (%)	0 - 10	10 - 20	20 – 40	40 – 45	45 – 60	60 - 100
Frequency density	0.8	1.0	1.5	4.4	2.8	0.4

(a) Construct a histogram for this data, and use it to calculate the modal mark. *(05 marks)*

(b) Plot a cumulative frequency curve, and use it to determine the;

(i) Number of student who scored at least a 50%.

(ii) Decile deviation. *(07 marks)*

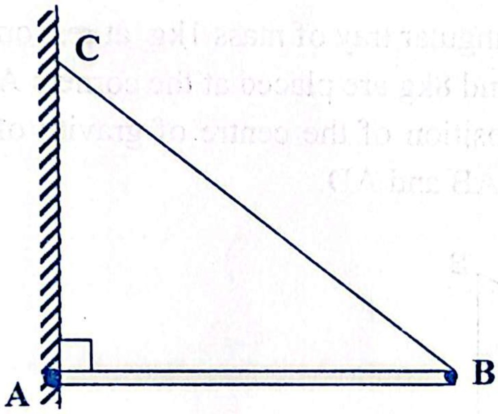
12. The heights of maize plants in a nursery bed are normally distributed with a mean of 16cm with a variance of $100cm^2$.

(a) Calculate the probability of obtaining a maize plant whose height is greater than 20cm. *(04 marks)*

(b) Twenty five plants are picked at random, find the probability that their mean height lies between 13cm and 19cm. *(04 marks)*

(c) How many plants should be picked at random before the probability that at least one of them has a height less than 16cm is greater than 0.9? *(04 marks)*

13.



AB is a uniform rod of mass 12kg, and of length 8m. The rod is smoothly hinged, to a vertical wall, at point A. A light inelastic string BC, of length 10m, keeps the rod at rest, in a horizontal position, as shown in the diagram above.

(a) Calculate the;

(i) Tension, **T** in the string. *(03 marks)*

(ii) Reaction at the hinge A. *(05 marks)*

(b) A boy of mass 40kg starts to walk along the rod from point A towards B. Given that the tension in the string cannot be exceed $\frac{3}{2}T$, calculate the distance the boy will walk before the string snaps. *(04 marks)*

14. (a) Use the trapezium rule with 6 intervals to evaluate; *(05 marks)*

$$\int_0^2 x\sin x\, dx, \qquad \textit{correct to 4 decimal places.}$$

(b) Given that $x = 1.60 \pm 0.005$ and $y = 4.8 \pm 0.05$; calculate the maximum error in computing y/x ; hence or otherwise state interval within which the exact value of y/x lies. *(07 marks)*

15. (a) A car of mass 1000 kg, travels a long a horizontal road against a constant resistance of 200 N. If the car develops a constant power of 30 kW, calculate the acceleration of the car at the instant when its speed is 30 ms^{-1}. ***(05 marks)***

(b) A steamer is initially 60km north of an observatory, O and travelling at 40 kmh^{-1} due N60°E. At the same instant, a boat capable of a maximum speed of 64 kmh^{-1} and initially 80 km East of O, sets off to intercept the steamer.

Calculate the two possible courses the boat can take, and determine the shortest time of interception. ***(07 marks)***

16. X is a continuous random variable whose probability density function is given as;

$$f(x) = \begin{cases} 2kx & ; \quad 0 \le x \le 2 \\ k(6-x) & ; \quad 2\cancel{4} \le x \le 6 \\ 0 & ; \quad elsewhere \end{cases}$$

(a) Sketch the graph of $f(x)$ and use it to determine the value of the constant *k*. ***(04 marks)***

(b) Find, $F(x)$, the cumulative distribution function of *X*, hence compute;

(i) $P(X > 5)$

(ii) the 80^{th} percentile of X. ***(08 marks)***

END